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- Let s₁ denote the largest size, and let b₁ denote the number of pieces of size s₁.
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We call a vector that fulfills the above constraint a configuration.



16.4 Advanced Rounding for Bin Packing

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$$\begin{array}{c|cccc} \min & & \sum_{j=1}^{N} x_j \\ \text{s.t.} & \forall i \in \{1 \dots m\} & \sum_{j=1}^{N} T_{ji} x_j & \geq & b_i \\ & \forall j \in \{1, \dots, N\} & x_j & \geq & 0 \\ & \forall j \in \{1, \dots, N\} & x_j & \text{integral} \end{array}$$



How to solve this LP?

later...



16.4 Advanced Rounding for Bin Packing

《聞》《園》《夏》 351/521 We can assume that each item has size at least 1/SIZE(I).



Sort items according to size (monotonically decreasing).

- Process items in this order; close the current group if size of items in the group is at least 2 (or larger). Then open new group.
- I.e., G_1 is the smallest cardinality set of largest items s.t. total size sums up to at least 2. Similarly, for G_2, \ldots, G_{r-1} .
- Only the size of items in the last group G_r may sum up to less than 2.



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- Round all items in a group to the size of the largest group member.
- Delete all items from group G_1 and G_r .
- For groups G_2, \ldots, G_{r-1} delete $n_i n_{i-1}$ items.
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Lemma 10 The number of different sizes in I' is at most SIZE(I)/2.

- Each group that survives (recall that Gy and Gy are deleted) has total size at least 2.
- Hence, the number of surviving groups is at most SIZE(/)/2...
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The total size of deleted items is at most $O(\log(SIZE(I)))$.

- The total size of items in G₁ and G₂ is at most 6 as a group that total size at most 3.
- Consider a group G_1 that has strictly more items than G_{1-1} . It discards $m_1 - m_{n-1}$ pieces of total size at most

- since the smallest piece has size at most $3/n_{\rm f}$.
- Summing over all 1 that have $n_l > n_{l-1}$ gives a bound of attained most

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- Consider a group G_i that has strictly more items than G_{i-1} .
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Algorithm 1 BinPack

- 1: **if** SIZE(I) < 10 **then**
- 2: pack remaining items greedily
- 3: Apply harmonic grouping to create instance I'; pack discarded items in at most $O(\log(\text{SIZE}(I)))$ bins.
- 4: Let x be optimal solution to configuration LP
- 5: Pack $\lfloor x_j \rfloor$ bins in configuration T_j for all j; call the packed instance I_1 .
- 6: Let I_2 be remaining pieces from I'
- 7: Pack I_2 via BinPack (I_2)



Analysis

$OPT_{LP}(I_1) + OPT_{LP}(I_2) \le OPT_{LP}(I') \le OPT_{LP}(I)$

Proof:

- Each piece surviving in I' can be mapped to a piece in I of no lesser size. Hence, $OPT_{D'}(I') \leq OPT_{D'}(I)$
- $[x_i]$ is feasible solution for f_i (even integral).
- $x_{ij} = \lfloor x_{ij} \rfloor$ is feasible solution for I_2 .



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16.4 Advanced Rounding for Bin Packing

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Each level of the recursion partitions pieces into three types

- 1. Pieces discarded at this level.
- **2.** Pieces scheduled because they are in I_1 .
- **3.** Pieces in *I*² are handed down to the next level.

Pieces of type 2 summed over all recursion levels are packed into at most OPT_{LP} many bins.

Pieces of type 1 are packed into at most

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We can show that $SIZE(I_2) \le SIZE(I)/2$. Hence, the number of recursion levels is only $O(\log(SIZE(I_{\text{original}})))$ in total.

configuration LP for J' is at most the number of constraints, which is the number of different sizes ($\leq SIZE(J)/2$). The total size of items in J_2 can be at most $\sum_{i=1}^{J} |z_i - |z_i|$ which is at most the number of non-zero entries in the solution to the configuration LP.



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- ► The total size of items in I₂ can be at most ∑_{j=1}^N x_j ⌊x_j⌋ which is at most the number of non-zero entries in the solution to the configuration LP.

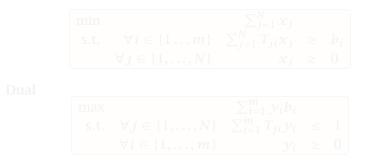


How to solve the LP?

Let T_1, \ldots, T_N be the sequence of all possible configurations (a configuration T_j has T_{ji} pieces of size s_i).

In total we have b_i pieces of size s_i .

Primal





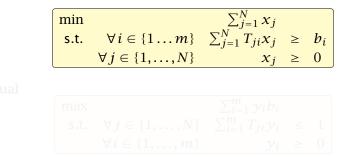
16.4 Advanced Rounding for Bin Packing

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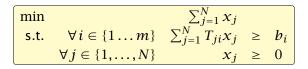
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Dual

$$\begin{array}{ll} \max & \sum_{i=1}^{m} y_i b_i \\ \text{s.t.} & \forall j \in \{1, \dots, N\} \quad \sum_{i=1}^{m} T_{ji} y_i \leq 1 \\ & \forall i \in \{1, \dots, m\} \quad y_i \geq 0 \end{array}$$



Suppose that I am given variable assignment y for the dual.

How do I find a violated constraint?

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I have to find a configuration T_j = (T_{j1}, \dots, T_{jm}) that
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But this is the Knapsack problem.



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We have FPTAS for Knapsack. This means if a constraint is violated with $1 + \epsilon' = 1 + \frac{\epsilon}{1-\epsilon}$ we find it, since we can obtain at least $(1 - \epsilon)$ of the optimal profit.

The solution we get is feasible for:

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$$\begin{array}{|c|c|c|c|c|} \min & (1+\epsilon')\sum_{j=1}^N x_j \\ \text{s.t.} & \forall i \in \{1 \dots m\} & \sum_{j=1}^N T_{ji} x_j \geq b_i \\ & \forall j \in \{1, \dots, N\} & x_j \geq 0 \end{array}$$

If the value of the computed dual solution (which may be infeasible) is z then

$\mathsf{OPT} \le z \le (1 + \epsilon')\mathsf{OPT}$

- The constraints used when computing 2 certify that the solution is feasible for DUAL'.
- Suppose that we drop all unused constraints in DUAL. We will compute the same solution feasible for DUAL'.
- Let DUAL" be DUAL without unused constraints.
- The dual to DUAL" is PRIMAD where we ignore variables for which the corresponding dual constraint has not been used.
- The optimum value for ${\sf PRIMAL}^{\prime\prime}$ is at most (1.4 ϵ^{\prime})OPT.
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- We can compute the corresponding solution in polytime.

This gives that overall we need at most

```
(1 + \epsilon')OPT<sub>LP</sub>(I) + O(\log^2(SIZE(I)))
```

bins.

We can choose $\epsilon' = \frac{1}{OPT}$ as $OPT \le \#$ items and since we have a fully polynomial time approximation scheme (FPTAS) for knapsack.



16.4 Advanced Rounding for Bin Packing

This gives that overall we need at most

```
(1 + \epsilon')OPT<sub>LP</sub>(I) + O(\log^2(SIZE(I)))
```

bins.

We can choose $\epsilon' = \frac{1}{OPT}$ as $OPT \le \#$ items and since we have a fully polynomial time approximation scheme (FPTAS) for knapsack.

