Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen (LEA)
Prof. Dr. Susanne Albers
Moritz Fuchs

Spring Semester 2014
Problem set 7
May 21, 2014

Online and approximation algorithms

Due May 28, 2014 before class!

Exercise 1 (10 points)

In this exercise we determine the mean number of bits $B_{MTF}(S)$ that are needed to encode a symbol of an arbitrary String S that was not generated using a memoryless source. As usual the alphabet is $\Sigma = \{x_1, ..., x_n\}$ and the number of bits that are needed to encode the natural number j is $f(j) = 1 + 2\lfloor \log j \rfloor$. Let N be the length of S and N_i the number of occurrences of a symbol x_i in S for $1 \leq i \leq n$. Show that

$$B_{MTF}(S) \leq \sum_{i=1}^{n} \frac{N_i}{N} f(\frac{N}{N_i})$$

Hint 1: Without loss of generality, assume that the initial list of symbols is ordered according to the first occurrence of each symbol, i.e. the first symbol of S is initially on the first position in the list. Determine for a fixed symbol x_i an upper bound on the number of bits that are necessary to encode *all* occurrences of x_i.

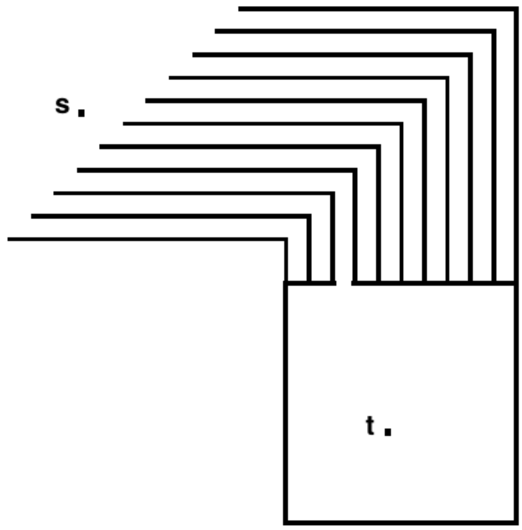
Hint 2: Use Jensen's inequality which states that for a concave function f, values $y_1, ..., y_m$ in the domain of f and real weights $w_1, ..., w_m$ s.t. $\sum_{i=1}^{m} w_i = 1$ holds

$$\sum_{j=1}^{m} w_j f(y_j) = f(\sum_{i=1}^{m} w_i y_i)$$

Exercise 2 (Room problem - 10 points)

In the lecture we saw an algorithm for the square room problem that achieves a competitive ratio of $\sqrt{n}$.
Modify the algorithm as well as its analysis for rooms of dimension $2N \times 2n$ where $N \neq n$.
The starting point is $s = (0, 0)$, the target $t = (N, n)$.

Exercise 3 (Non-convex obstacles - 10 points)



Recall the navigation problem from the lecture. A robot is sitting at point s and wants to reach point t as efficiently as possible. However he does not have a map of its surroundings and can only sense collisions with obstacles. In the lecture we assumed all obstacles to be convex, now we want to investigate the navigation problem using non-convex obstacles. In the above setting (figure due to [1]) we have exactly one non-convex obstacle between s and t. This obstacle has 10 passages, only one of which leads to t. We say that each line segment consists of exactly four vertices and correspond to the set of all vertices by V. (e.g. the above obstacle has 70 vertices)

(a) We can generalize the obstacle above to have k passages instead of 10 by adding passages that do not lead to t. Show that any maze generated that way has exactly $(|V| - 10)/6$ passages.

(b) Show that no randomized navigation algorithm can achieve a better ratio than $(|V|-10)/6$.

Exercise 4 (Find Mr.X! - 10 points)

Image yourself standing on a corridor looking for Mr.X. The corridor has infinitely many doors in both directions and each door is marked with a name. In each step you are allowed to walk to the next door in either direction for cost 1. Upon entering the facility you were given a fair coin as well as the advice to use it wisely.

Develop a 7-competitive strategy for finding Mr.X and prove its competitiveness.