Technische Universität München
Fakultät für Informatik
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Winter Semester 2014/15
Problem Set 11
January 12th, 2015

Parallel Algorithms

Due date: January 19th, 2014 **before** *class!*

Problem 1 (10 Points)

Consider greedy routing on the hypercube with random intermediate locations, i.e. if a packet at node i has to be sent to node j, the routing protocol first sends it to a random node r, then sends it from r to j. In this case, routing paths can be as long as $2d$, where d is the dimension of the hypercube. Now, consider the following variation: for every packet and its random intermediate location r, we compare the lengths of the greedy paths $i \to \cdots \to r \to \cdots \to j$ and $i \to \cdots \to \overline{r} \to \cdots \to j$ (with $\overline{r}$ having the encoding of r with all bits flipped), and route the packet along the shorter path.
Show that the routing paths are shorter in the worst case.

Problem 2 (20 Points)

A *packing* problem consists of routing any collection of $m \leq n$ packets contained in level $\log n$ of a $\log n$-dimensional butterfly to the first m nodes in level 0 of the butterfly such that the relative order of the packets remains unchanged.

1. Consider removing all nodes (and incident edges) from the leftmost or rightmost level of a butterfly, respectively. Which structure have the remaining networks?
2. It is possible that a processor that contains a packet in a packing problem may not know the correct destination for the packet. How can you figure out the correct destinations for the packets with only two runs through the butterfly?
   Hint: Use a parallel prefix.
3. Show that after the first step of the greedy packing protocol, there are no collisions, i.e. there are no two packets sent to the same node.
   Hint: Consider two neighboring packets. What is the difference in their corresponding destinations?
   Keep in mind that for this case, you have a reversed butterfly at hand, as seen in Fig. 1. Would this work for a normal butterfly and why (not)?
4. Complete the proof that there are no collisions on any level by induction.

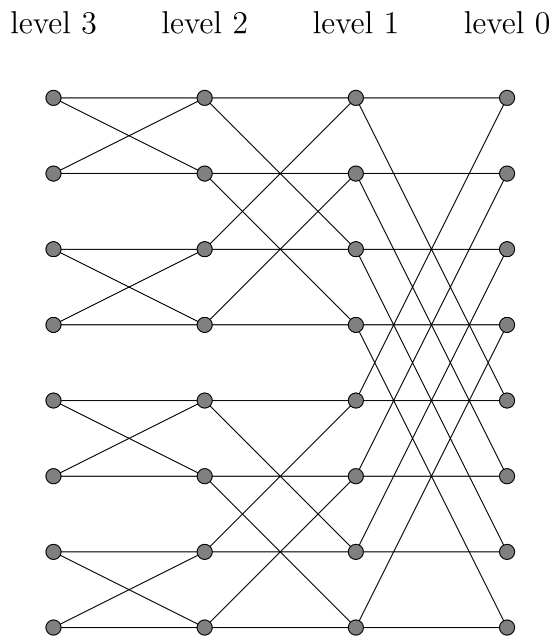

Figure 1: reverse butterfly

Problem 3 (10 Points)

A *spreading* problem consists of routing a **contiguous** set of $m \leq n$ packets contained in the first m nodes of level 0 of a $\log n$-dimensional butterfly to any collection of m destinations at level $\log n$ such that the relative order of the packets remains unchanged.
A *monotone* routing problem is one where the relative order of the packets remains unchanged.

1. Show how to greedily route any spreading permutation on a butterfly with a congestion of 1.

2. Show how to greedily route any monotone permutation with a congestion of 1. *Hint:* You may traverse the (reverse) butterfly several times for this.