Prof. Dr. Susanne Albers
Dr. Suzanne van der Ster
Dario Frascaria
Lehrstuhl für Theoretische Informatik
Institut für Informatik
Technische Universität München

Fall Semester
January 11, 2016

Randomized Algorithms

Exercise Sheet 11

Due: January 18, 2016
at 10:15, in class

Exercise 11.1 (10 points)
Consider the bit-fixing routing algorithm for routing a permutation on the n-cube. Suppose that n is even. Write each source node s as the concatenation of two binary strings a_s and b_s, each of length $n/2$. Let the destination the packet from s be the concatenation of b_s and a_s. Show that this permutation causes the bit-fixing routing algorithm to take $\Omega(\sqrt{N})$ steps, where $N = 2^n$.

Exercise 11.2 (10 points)
Consider the following variant of the bit-fixing algorithm for permutation routing on the n-cube. Instead of fixing the bits in order from 1 to n, each packet chooses a random order (independently from the other packets) and fixes the bits in that order.

Show that there is a permutation such that the expected number of steps taken by this algorithm is $2^{\Omega(n)}$.
Hint: Consider the same permutation as in the previous exercise. That is, each packet has a source node $(a_1, a_2, \ldots, a_{n/2}, b_1, b_2, \ldots, b_{n/2})$ *and a destination node* $(b_1, b_2, \ldots, b_{n/2}, a_1, a_2, \ldots, a_{n/2})$*. Then, show that the expected number of packets that pass through the node* $(0, 0, \ldots, 0)$ *is* $2^{\Omega(n)}$*.*
You may use the inequality $\left(\frac{n}{k}\right)^k \leq \binom{n}{k} \leq \left(\frac{en}{k}\right)^k$ *which holds for any pair of positive integers* $k \leq n$*.*

Exercise 11.3 (15 points)
Assume that we use the bit-fixing routing algorithm for the n-cube network to route a total of up to $p2^n$ packets, where each node is the source of no more than p packets and each node is the destination of no more than p packets.

(a) Give a high-probability bound on the run time of the algorithm.

(b) Give a high-probability bound on the maximum number of packets at any node at any step of the execution of the routing algorithm.

Exercise 11.4 (5 points)
Consider the global wiring problem and a fractional solution $\hat{x}$ of the linear programming relaxation of this problem. Give a simple rounding procedure that obtains rounded solutions $\bar{x}$ from $\hat{x}$ so that $w_s \leq 2w_0$, where w_0 is the optimum solution for our restricted class of one-bend routes.