Prof. Dr. Susanne Albers
Dr. Suzanne van der Ster
Dario Frascaria
Lehrstuhl für Theoretische Informatik
Fakultät für Informatik
Technische Universität München

Problem set 12
July 1, 2016
Summer Semester 2016

Online and Approximation Algorithms

Due July 8, 2016 before 10:00

Exercise 1 (2-SAT - 10 points)

Consider the *2-SAT* problem. In this problem we are given a boolean formula in *Conjunctive Normal Form* where all clauses contain two literals (2CNF) and have to decide whether the formula is satisfiable or not.

(a) Describe a method of encoding the given *2CNF*-formula as a directed graph.

(b) Develop an algorithm that decides in polynomial time whether the given formula can be satisfied or not.

(c) Develop an algorithm that returns a satisfying assignment if it exists and show that it runs in polynomial time as well.

Exercise 2 (Max-NAE 3-SAT - 10 points)

We consider the *Max-NAE 3-SAT* (Max Not-All-Equal 3-SAT) problem. In this problem we are given a *3CNF* formula and our goal is to find a variable assignment, such that the number of clauses containing at least one true **and** one false literal is maximized.
Develop a randomized approximation algorithm for *Max-NAE 3-SAT* with an approximation ratio of $\frac{3}{4}$.

Exercise 3 (Set Cover and Vertex Cover - 10 points)

Recall that in the Vertex Cover problem, one has to select a subset of vertices from a graph G such that every edge in G is incident to at least one vertex of this set. The goal is to find such a set of minimum cost.
Let 2SC be the Set Cover problem restricted to instances where each item appears in at most two sets.
Show that 2SC is equivalent to the Vertex Cover problem with arbitrary costs. That is, show that an instance of the Vertex Cover problem can be reduced to an instance of 2SC and that for both problems the same approximation ratio can be obtained.

Exercise 4 (ILP Formulation - 10 points)

Give an ILP formulation for the Bin Packing problem where our objective is to pack items $1, 2, \ldots, n$ with volumes $v_1, v_2, \ldots, v_n$ in a minimum number of bins of unit size.
Declare your variables and explain your constraints.