

7.5 (a, b) -trees

Definition 1

For $b \geq 2a - 1$ an (a, b) -tree is a search tree with the following properties

1. all leaves have the same distance to the root
2. every internal non-root vertex v has at least a and at most b children
3. the root has degree at least 2 if the tree is non-empty
4. the internal vertices do not contain data, but only keys (external search tree)
5. there is a special dummy leaf node with key-value ∞

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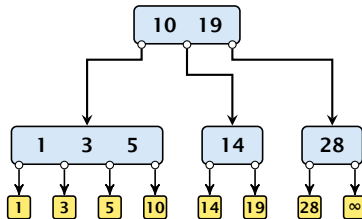
Each internal node v with $d(v)$ children stores $d - 1$ keys k_1, \dots, k_{d-1} . The i -th subtree of v fulfills

$$k_{i-1} < \text{key in } i\text{-th sub-tree} \leq k_i ,$$

where we use $k_0 = -\infty$ and $k_d = \infty$.

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Example 2



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Variants

- ▶ The dummy leaf element may not exist; it only makes implementation more convenient.
- ▶ Variants in which $b = 2a$ are commonly referred to as B -trees.
- ▶ A B -tree usually refers to the variant in which keys and data are stored at internal nodes.
- ▶ A B^+ tree stores the data only at leaf nodes as in our definition. Sometimes the leaf nodes are also connected in a linear list data structure to speed up the computation of successors and predecessors.
- ▶ A B^* tree requires that a node is at least $2/3$ -full as opposed to $1/2$ -full (the requirement of a B -tree).

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Lemma 3

Let T be an (a, b) -tree for $n > 0$ elements (i.e., $n + 1$ leaf nodes) and height h (number of edges from root to a leaf vertex). Then

1. $2a^{h-1} \leq n + 1 \leq b^h$
2. $\log_b(n + 1) \leq h \leq 1 + \log_a\left(\frac{n+1}{2}\right)$

Proof.

The root has degree at least 2, and all other nodes

have degree at least 1. This gives that $n + 1 \leq b^h$.

Since all nodes have degree at most a , we have

$n + 1 \geq 2a^{h-1}$. This gives $h \leq 1 + \log_a\left(\frac{n+1}{2}\right)$.



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- ▶ Analogously, the degree of any node is at most b and, hence, the number of leaf nodes at most b^h .



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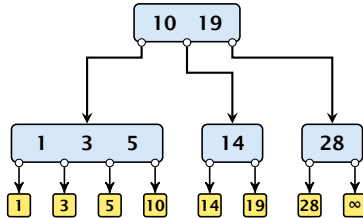
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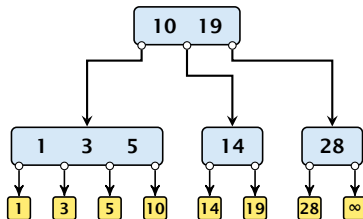


Search



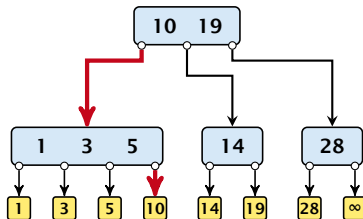
Search

Search(8)



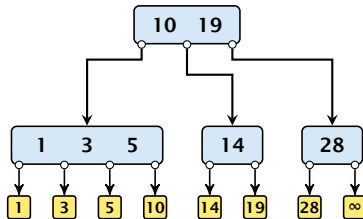
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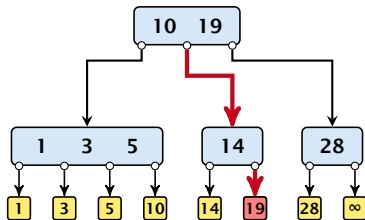
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Search(19)

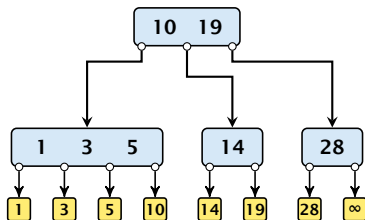


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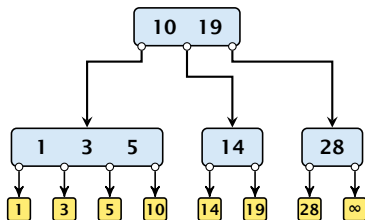


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Time: $\mathcal{O}(b \cdot h) = \mathcal{O}(b \cdot \log n)$, if the individual nodes are organized as linear lists.

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Insert element x :

- ▶ Follow the path as if searching for $\text{key}[x]$.
- ▶ If this search ends in leaf ℓ , insert x before this leaf.
- ▶ For this add $\text{key}[x]$ to the key-list of the last internal node v on the path.
- ▶ If after the insert v contains b nodes, do $\text{Rebalance}(v)$.

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Rebalance(v):

- ▶ Let k_i , $i = 1, \dots, b$ denote the keys stored in v .
- ▶ Let $j := \lfloor \frac{b+1}{2} \rfloor$ be the middle element.
- ▶ Create two nodes v_1 , and v_2 . v_1 gets all keys k_1, \dots, k_{j-1} and v_2 gets keys k_{j+1}, \dots, k_b .
- ▶ Both nodes get at least $\lfloor \frac{b-1}{2} \rfloor$ keys, and have therefore degree at least $\lfloor \frac{b-1}{2} \rfloor + 1 \geq a$ since $b \geq 2a - 1$.
- ▶ They get at most $\lceil \frac{b-1}{2} \rceil$ keys, and have therefore degree at most $\lceil \frac{b-1}{2} \rceil + 1 \leq b$ (since $b \geq 2$).
- ▶ The key k_j is promoted to the parent of v . The current pointer to v is altered to point to v_1 , and a new pointer (to the right of k_j) in the parent is added to point to v_2 .
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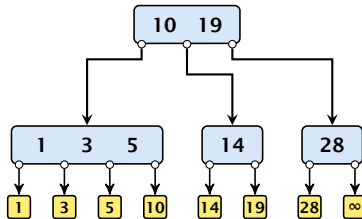
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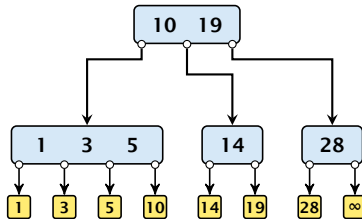
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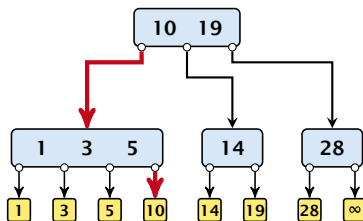
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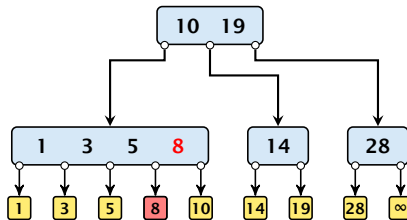
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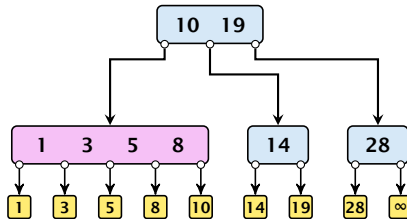
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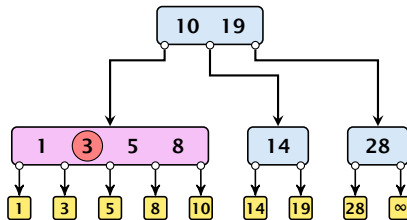
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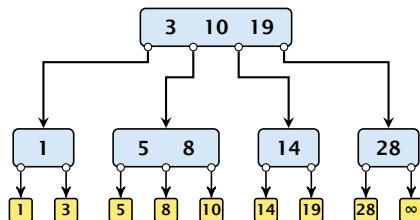


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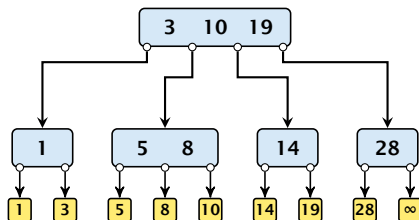


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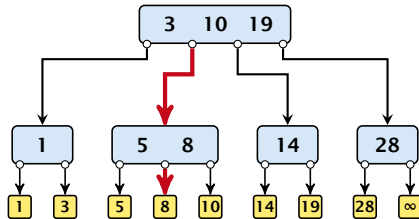
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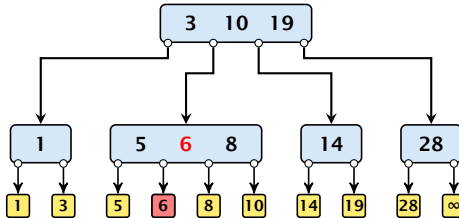
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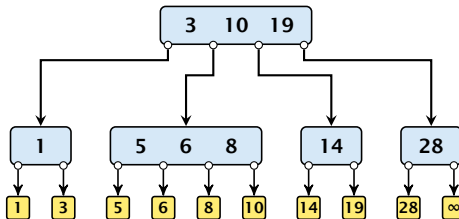
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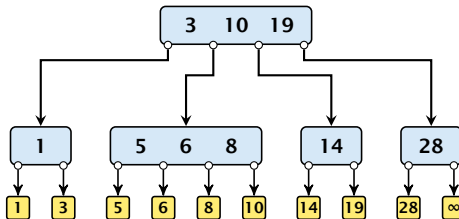
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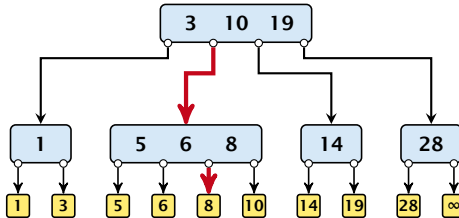
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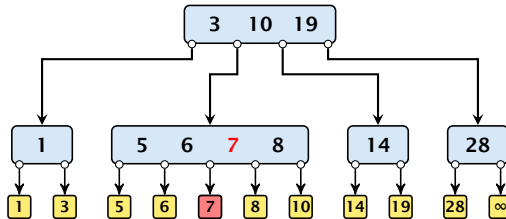
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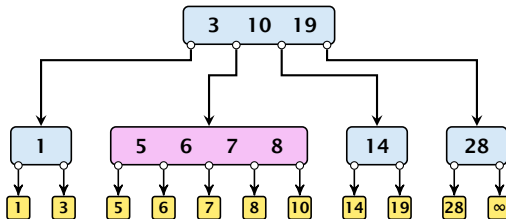
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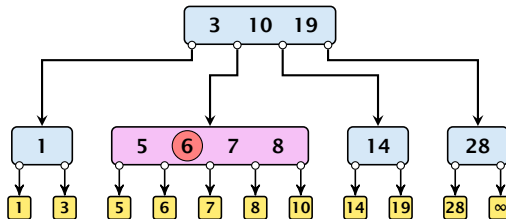
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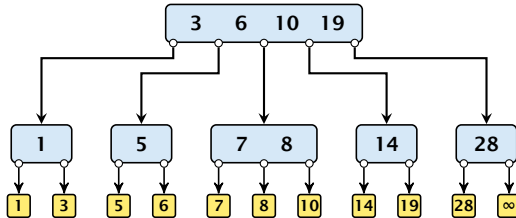
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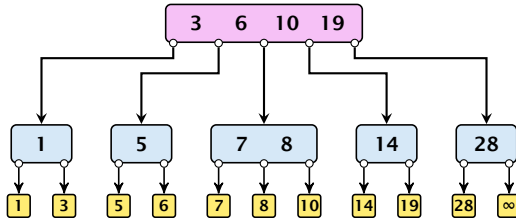
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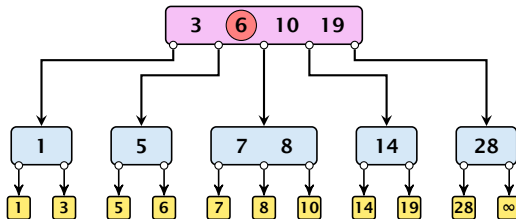
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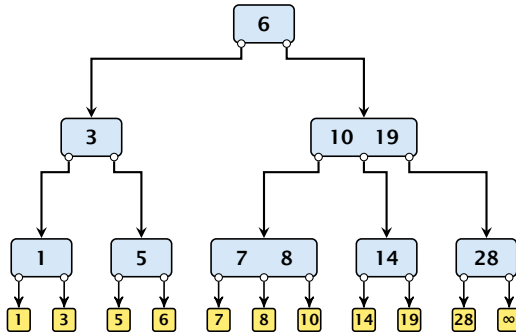
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Delete element x (pointer to leaf vertex):

- ▶ Let v denote the parent of x . If $\text{key}[x]$ is contained in v , remove the key from v , and delete the leaf vertex.
- ▶ Otherwise delete the key of the predecessor of x from v ; delete the leaf vertex; and replace the occurrence of $\text{key}[x]$ in internal nodes by the predecessor key. (Note that it appears in exactly one internal vertex).
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- ▶ Otherwise delete the key of the predecessor of x from v ; delete the leaf vertex; and replace the occurrence of $\text{key}[x]$ in internal nodes by the predecessor key. (Note that it appears in exactly one internal vertex).
- ▶ If now the number of keys in v is below $a - 1$ perform $\text{Rebalance}'(v)$.

Delete

Rebalance' (v):

- ▶ If there is a neighbour of v that has at least a keys take over the largest (if right neighbor) or smallest (if left neighbour) and the corresponding sub-tree.
- ▶ If not: merge v with one of its neighbours.
- ▶ The merged node contains at most $(a - 2) + (a - 1) + 1$ keys, and has therefore at most $2a - 1 \leq b$ successors.
- ▶ Then rebalance the parent.
- ▶ During this process the root may become empty. In this case the root is deleted and the height of the tree decreases.

Delete

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Delete

Rebalance' (v):

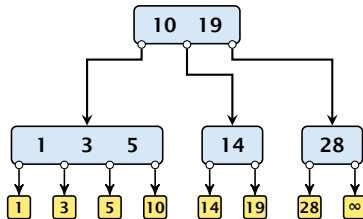
- ▶ If there is a neighbour of v that has at least a keys take over the largest (if right neighbor) or smallest (if left neighbour) and the corresponding sub-tree.
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Delete

Rebalance' (v):

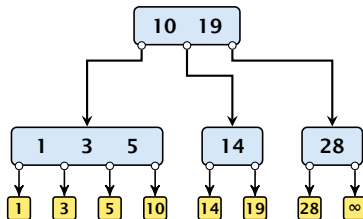
- ▶ If there is a neighbour of v that has at least a keys take over the largest (if right neighbor) or smallest (if left neighbour) and the corresponding sub-tree.
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- ▶ The merged node contains at most $(a - 2) + (a - 1) + 1$ keys, and has therefore at most $2a - 1 \leq b$ successors.
- ▶ Then rebalance the parent.
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Delete



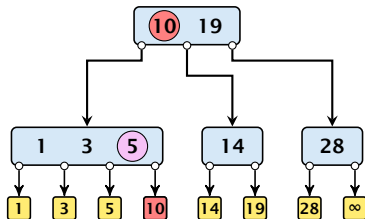
Delete

Delete(10)



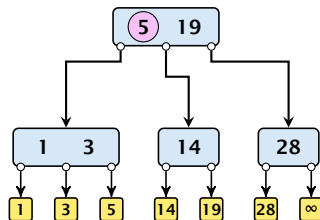
Delete

Delete(10)

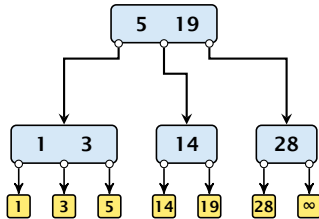


Delete

Delete(10)

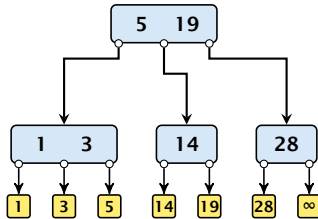


Delete



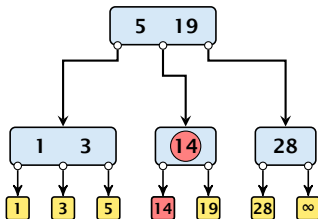
Delete

Delete(14)



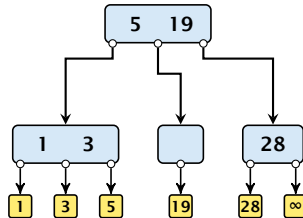
Delete

Delete(14)



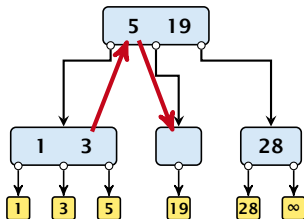
Delete

Delete(14)



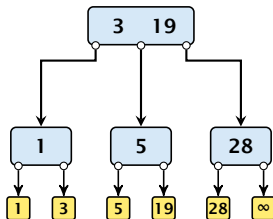
Delete

Delete(14)

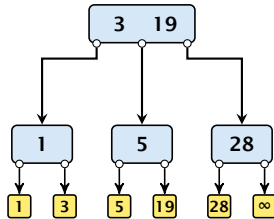


Delete

Delete(14)

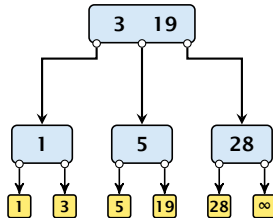


Delete



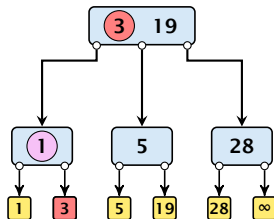
Delete

Delete(3)



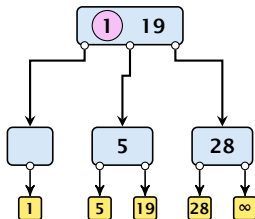
Delete

Delete(3)



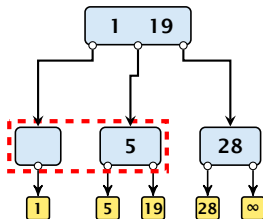
Delete

Delete(3)



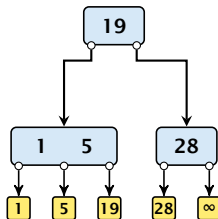
Delete

Delete(3)

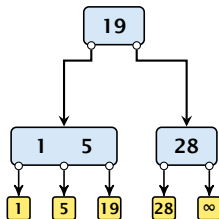


Delete

Delete(3)

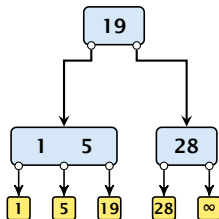


Delete



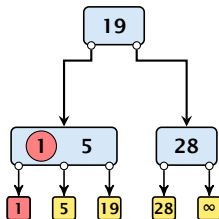
Delete

Delete(1)



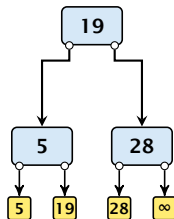
Delete

Delete(1)

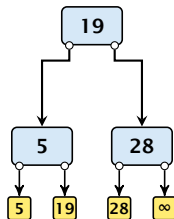


Delete

Delete(1)

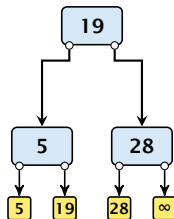


Delete



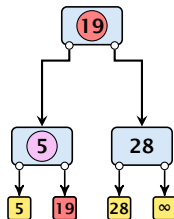
Delete

Delete(19)



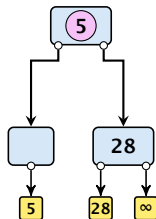
Delete

Delete(19)



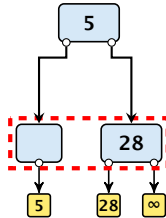
Delete

Delete(19)



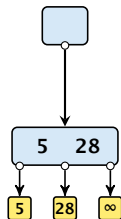
Delete

Delete(19)



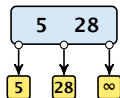
Delete

Delete(19)



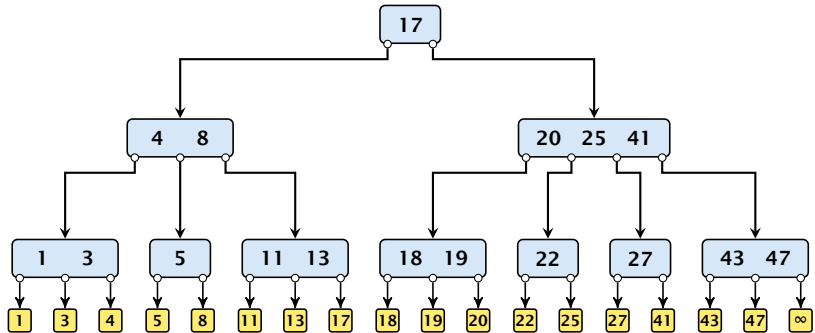
Delete

Delete(19)



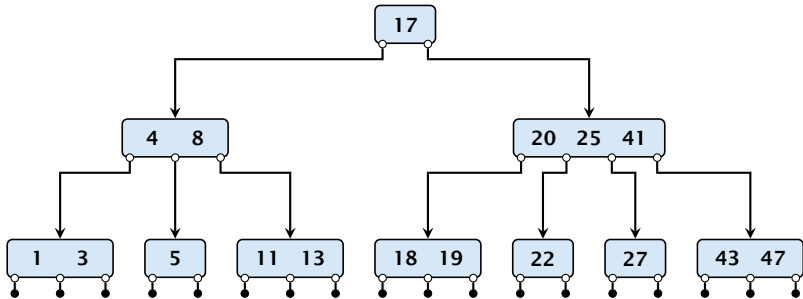
(2, 4)-trees and red black trees

There is a close relation between red-black trees and (2,4)-trees:



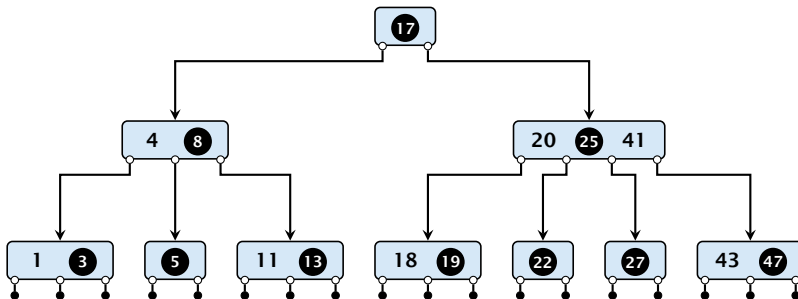
(2, 4)-trees and red black trees

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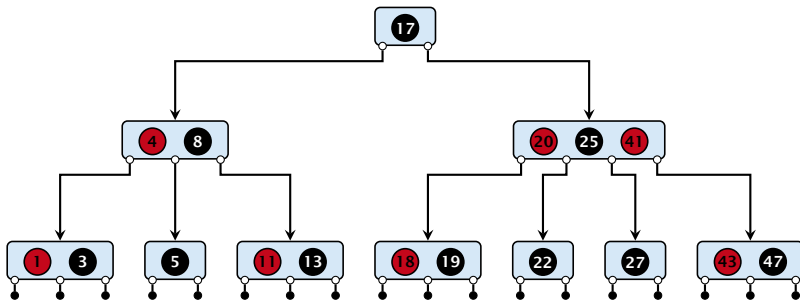
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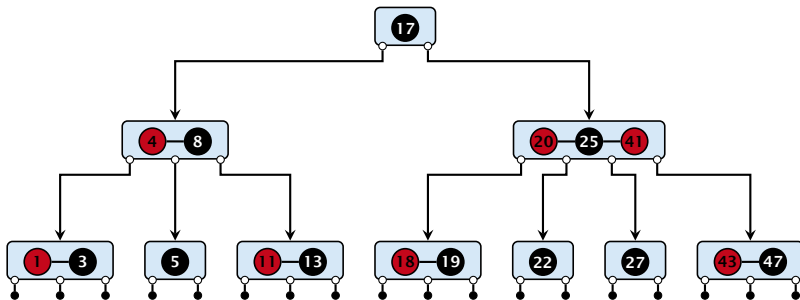
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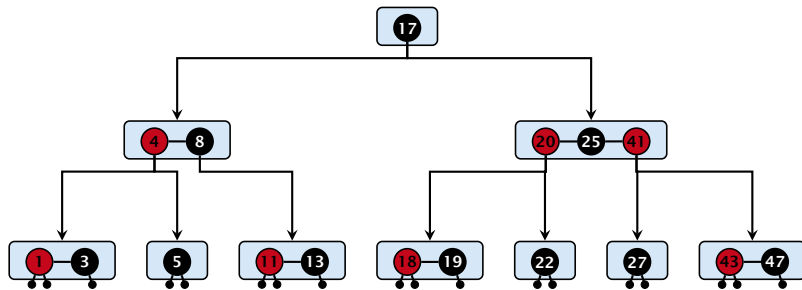
(2, 4)-trees and red black trees

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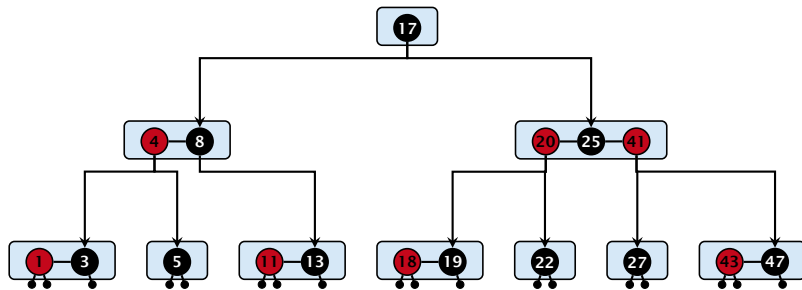
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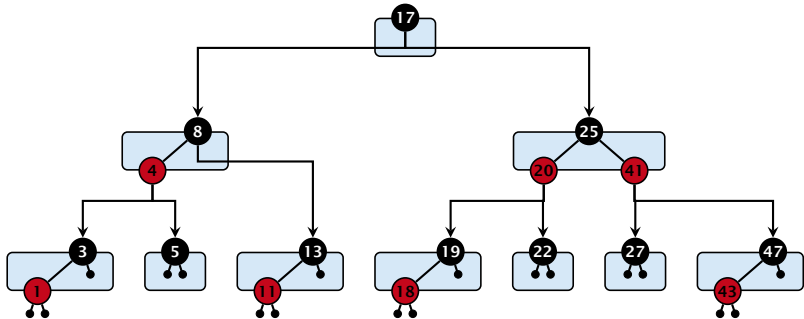
(2, 4)-trees and red black trees

There is a close relation between red-black trees and (2,4)-trees:



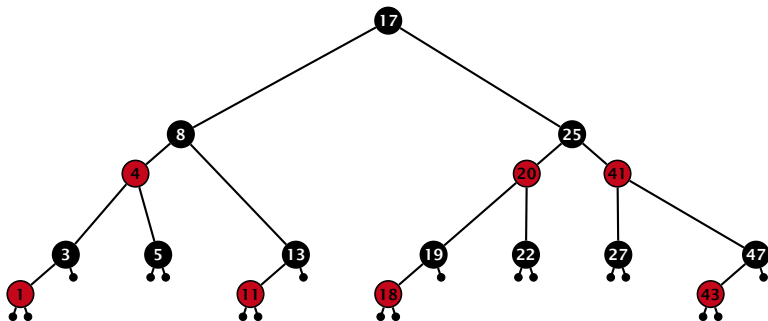
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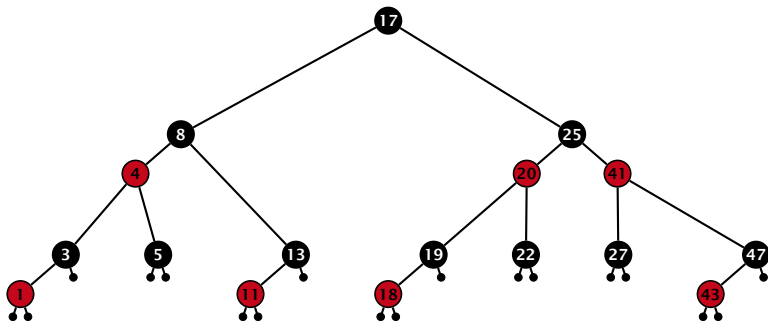
(2, 4)-trees and red black trees

There is a close relation between red-black trees and (2, 4)-trees:



(2, 4)-trees and red black trees

There is a close relation between red-black trees and (2, 4)-trees:



Note that this correspondence is not unique. In particular, there are different red-black trees that correspond to the same (2, 4)-tree.