## **Online and Approximation Algorithms**

Due November 13, 2017 at 10:00

## Exercise 1 (List Update, Paid exchanges – 10 points)

- (a) Consider the three-element list with the following initial configuration: (A, B, C). Prove that the optimal offline cost to serve the request sequence C, B, C, B is 8. Now consider an optimal offline algorithm without paid exchanges and prove that it pays 9 to serve the same sequence.
- (b) Show that any list accessing algorithm can be transformed into an algorithm that only uses paid transactions without increasing the cost.

## Exercise 2 (RMARK – 10 points)

Consider a memory system with fast memory of size k. RMARK is the randomized online paging algorithm that works as follows.

Initially, all pages are unmarked. Whenever a page is requested it becomes marked. When a page is brought into fast memory, it replaces a randomly and uniformly chosen page from the set of unmarked pages that are in fast memory. When all pages in fast memory are marked and a page fault occurs, all pages become unmarked. Prove that RMARK is  $H_k$ -competitive against oblivious adversaries when the total number of pages equals k+1.