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- Implementing and testing on representative inputs
  - How do you choose your inputs?
  - May be very time-consuming.
  - Very reliable results if done correctly.
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Suppose n numbers from the interval $\{1, \dots, N\}$ have to be sorted. In this case we usually say that the input length is n instead of e.g. $n \log N$, which would be the number of bits required to encode the input.

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Model of Computation

How to measure performance

- Calculate running time and storage space etc. on a simplified, idealized model of computation, e.g. Random Access Machine (RAM), Turing Machine (TM), ...
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Version is often easier, but focusing on one type of operation makes it more difficult to obtain meaningful results.

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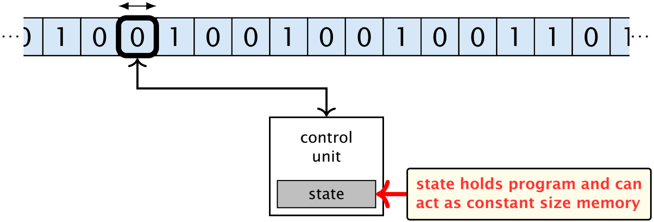
Turing Machine

- Very simple model of computation.
- Only the “current” memory location can be altered.
- Very good model for discussing computabiliy, or polynomial vs. exponential time.
- Some simple problems like recognizing whether input is of the form xx, where x is a string, have quadratic lower bound.

$\Rightarrow$ **Not a good model for developing efficient algorithms.**



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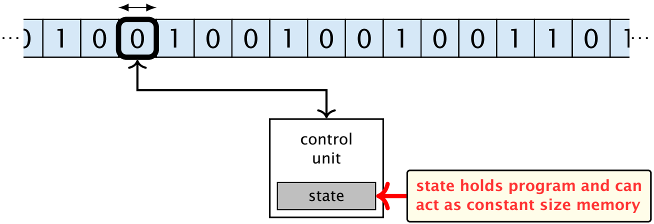

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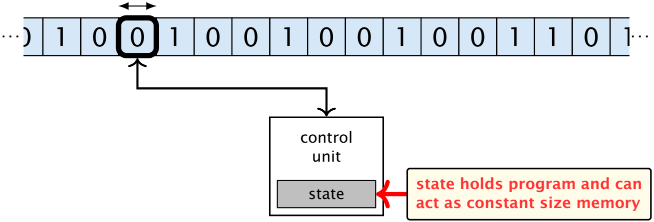

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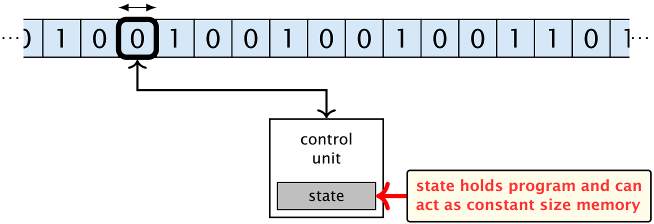

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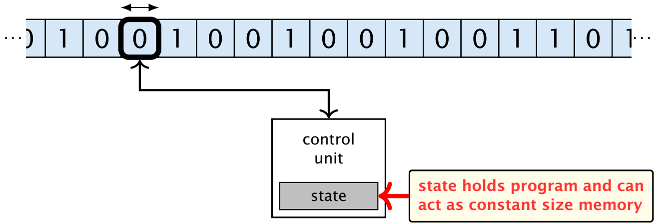
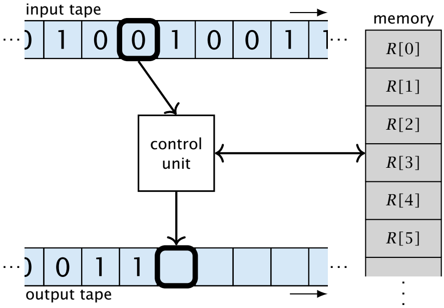


Random Access Machine (RAM)

- Input tape and output tape (sequences of zeros and ones; unbounded length).
- Memory unit: infinite but countable number of registers $R[0], R[1], R[2], \ldots$.
- Registers hold integers.
- Indirect addressing.



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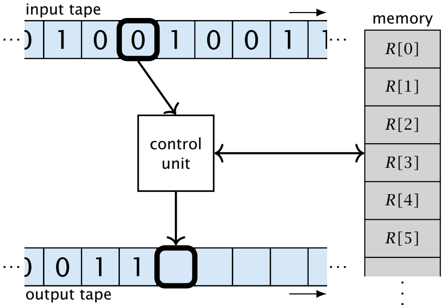

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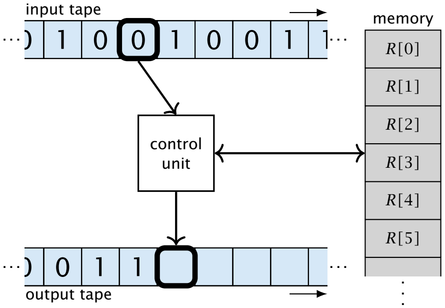

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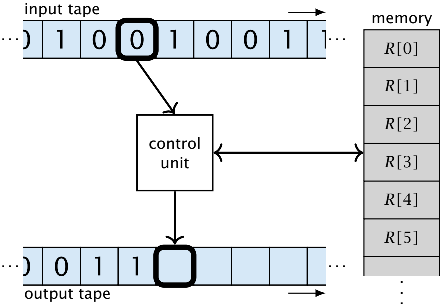

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Operations

- input operations (input tape $\to R[i]$)
  - READ i
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  - $R[j] := R[i]$
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- **indirect** addressing
  - $R[j] := R[R[i]]$
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- branching (including loops) based on comparisons
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    jumps to position x in the program;
    sets instruction counter to x;
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  - jumpz x $R[i]$
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  - jumpi i
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- arithmetic instructions: $+, -, \times, /$
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- uniform cost model
  Every operation takes time 1.

- logarithmic cost model
  The cost depends on the content of memory cells:
  - The time for a step is equal to the largest operand involved;
  - The storage space of a register is equal to the length (in bits) of the largest value ever stored in it.

Bounded word RAM model: cost is uniform but the largest value stored in a register may not exceed 2^w, where usually $w = \log_2 n$.

The latter model is quite realistic as the word-size of a standard computer that handles a problem of size n must be at least $\log_2 n$ as otherwise the computer could either not store the problem instance or not address all its memory.

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Model of Computation

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  Every operation takes time 1.
- logarithmic cost model
  The cost depends on the content of memory cells:
  - The time for a step is equal to the largest operand involved;
  - The storage space of a register is equal to the length (in bits) of the largest value ever stored in it.

Bounded word RAM model: cost is uniform but the largest value stored in a register may not exceed 2^w, where usually $w = \log_2 n$.

The latter model is quite realistic as the word-size of a standard computer that handles a problem of size n must be at least $\log_2 n$ as otherwise the computer could either not store the problem instance or not address all its memory.

4 Modelling Issues

Example 2

Algorithm 1 RepeatedSquaring(n)

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r ← 2;
for i = 1 → n do
    r ← r^2
return r
```

running time

space requirement

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There are different types of complexity bounds:

- best-case complexity:

$$C_{bc}(n) := \min\{C(x) \mid |x| = n\}$$

  Usually easy to analyze, but not very meaningful.

- worst-case complexity:

$$C_{wc}(n) := \max\{C(x) \mid |x| = n\}$$

  Usually moderately easy to analyze; sometimes too pessimistic.

- average case complexity:

$$C_{avg}(n) := \frac{1}{|I_n|} \sum_{|x|=n} C(x)$$

  more general: probability measure μ

$$C_{avg}(n) := \sum_{x \in I_n} \mu(x) \cdot C(x)$$

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